Bernstein wave instability in collisionless shocks

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Letters to the Editor

Bernstein wave instability in collisionless shocks

Abstract. The growth rate for the instability of Bernstein waves in perpendicular collisionless shocks is calculated including pressure gradient effects. For typical experimental conditions the growth rate can be as much as an order of magnitude greater than that predicted by previous calculations.

Recently a number of papers have been concerned with the instability of Doppler-shifted Bernstein waves in collisionless shocks propagating perpendicular to an applied magnetic field. Wong (1970) and Gary and Sanderson (1970) obtained an analytic expression for the growth rate of the instability for the case of cold ions. Lashmore-Davies (1970) pointed out that the instability arises because Bernstein waves can have negative energy in the presence of a drift current and he deduced that instability can occur for $T_i \gtrsim T_e$. Gary and Biskamp (1971) have given a more accurate expression for the growth rate in this case and discussed the significance of this instability for collisionless shock wave experiments. The current (or equivalently the Doppler shift of the Bernstein modes) is caused by the drift of electrons in the plane of the shock and perpendicular to the magnetic field; the ions have a Larmor radius much greater than the shock thickness and therefore suffer no drift. The electron drift velocity is the resultant of the $\boldsymbol{E}\times\boldsymbol{B}$ drift (due to the voltage jump across the shock) and the pressure gradient drift; this letter reports the results of an extension of the theory to include the ∇p_e drift which has previously been ignored.

We use the same geometry as in Gary and Sanderson (1970) so that in the rest frame of the shock the steady-state fields and electron pressure are given by

$$\boldsymbol{E} = -E_0\hat{\boldsymbol{x}} \qquad \boldsymbol{B} = B_0(1+\epsilon x)\hat{\boldsymbol{z}} \qquad p_e = p_0(1+\Delta x)$$

where E_0, ϵ and Δ are positive quantities. Following Krall (1968) we take the zero-order electron distribution function as

$$f_e^{(0)} = \frac{n_0}{(2\pi v_e{}^2)^{3/2}}\left\{1+\left(x-\frac{v_y}{\Omega_e}\right)\left(\epsilon'+\frac{\delta v^2}{2v_e{}^2}\right)\right\}\exp\left(-\frac{(\boldsymbol{v}-\boldsymbol{v}_0)^2}{2v_e{}^2}\right) \tag{1}$$

where $v_e{}^2 = (T_e/m_e)$, $\Omega_e = (eB_0/m_ec)$, and $\boldsymbol{v}_0 = (cE_0/B_0)\hat{\boldsymbol{y}}$ is the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity. Then from the usual definitions of pressure and temperature it follows that $\Delta = \epsilon'+5\delta/2$ and $\delta = \mathrm{d}(\lg T_e)/\mathrm{d}x$.

The drift current is given by

$$j = -n_0 e\boldsymbol{v}_d = -n_0 e\int \boldsymbol{v} f_e{}^{(0)}\mathrm{d}\boldsymbol{v} = \frac{c}{4\pi}\nabla\times\boldsymbol{B}$$

so that the net drift velocity

$$v_{\rm d} = v_0 - v_{\rm p} = \frac{\epsilon B_0 c}{4\pi n_0 e} \equiv \frac{2\bar{v}_B}{\beta_{\rm e}} \tag{2}$$

where $v_{\rm p} = (\Delta v_{\rm e}^2/\Omega_{\rm e})$ is the $\nabla p_{\rm e}$ drift and $\bar{v}_B = (\epsilon v_{\rm e}^2/\Omega_{\rm e})$ is an average ∇B drift. Previous calculations have ignored $v_{\rm p}$ and set $v_0 = v_{\rm d}$. In strong shocks v_0 and $v_{\rm p}$ can be much greater than $v_{\rm d}$ so the inclusion of pressure gradient effects is clearly desirable.

We now consider a small perturbation $f^{(1)}$ of the steady-state distribution $f_{\rm e}^{(0)}$ giving rise to electrostatic waves with wave vector $\boldsymbol{k}$ perpendicular to $\boldsymbol{B}$ and parallel to $\boldsymbol{v}_{\rm d}$. The linear dispersion relation is obtained in the usual way (Krall 1968), with the result

$$1 + K_{\rm i} + K_{\rm e} = 0 \tag{3}$$

where

$$K_{\cdot} = -\frac{k_{\rm e}^2 T_{\rm e}}{2k^2 T} Z'\left(\frac{\omega}{kv\sqrt{2}}\right) \tag{4}$$

$$K_{\rm e} = \frac{k_{\rm e}^2}{k^2}\left[1 - \sum_{l=-\infty}^{\infty} \frac{{\rm e}^{-x}}{(\omega - kv_0 + l\Omega_{\rm e})}\{(\omega - kv_{\rm d} - kv_T)I_l(x) - kv_T x(I_l(x) - I_l'(x))\}\right] \tag{5}$$

$$x = k^2 v_{\rm e}^2/\Omega_{\rm e}^2 \qquad k_{\rm e}^2 = (4\pi n_0 e^2/T_{\rm e}) \qquad v_{\rm i}^2 = (T_{\rm i}/m_{\rm i}).$$

In equation (5) $v_T = (\delta v_{\rm e}^2/\Omega_{\rm e})$ is the temperature gradient drift velocity, and $I_l(x)$ is the modified Bessel function of order l. For simplicity we here omit terms in $K_{\rm e}$ arising from the magnetic field gradient; this does not affect our results significantly.

To investigate the instability of the Doppler-shifted Bernstein mode with $\omega_R - kv_0 \simeq -l\Omega_{\rm e}$ we keep just the resonant term of the infinite sum in equation (5); then equation (3) becomes for $x \gg 1$

$$1 - \frac{k_{\rm e}^2 T_{\rm e}}{2k^2 T_{\rm i}} Z'\left(\frac{\omega}{kv_{\rm i}\sqrt{2}}\right) + \frac{k_{\rm e}^2}{k^2}\left(1 - \frac{\omega - k(v_{\rm d} + \frac{3}{2}v_T)}{(\omega - kv_0 + l\Omega_{\rm e})(2\pi x)^{1/2}}\right) = 0. \tag{6}$$

Following Lashmore-Davies (1970) we look for a solution with $\omega = kv_{\rm i}\sqrt{2}$ and find

$$\frac{\gamma}{\Omega e} \simeq \frac{0{\cdot}368\, T_{\rm e}(v_{\rm d} + \frac{3}{2}v_T - v_{\rm i}\sqrt{2})}{T_{\rm i} v_{\rm e}\sqrt{2}\{(1 + k^2/k_{\rm e}^2 - 0{\cdot}075\, T_{\rm e}/T_{\rm i})^2 + 0{\cdot}425\,(T_{\rm e}/T_{\rm i})^2\}} \tag{7}$$

where we have written the result in a form comparable with the expression given by Gary and Biskamp (1971). Thus inclusion of the pressure gradient increases γ by a factor

$$R = 1 + \frac{3v_T}{2(v_{\rm d} - v_{\rm i}\sqrt{2})}. \tag{8}$$

(In equation (6) of Gary and Biskamp (1971) $v_0 \equiv v_{\rm d}$ since $v_{\rm p} = 0$). The magnitude of R varies through the shock since $v_{\rm d} \propto B/n$ is approximately constant, while $v_T \propto T_{\rm e}/B$ increases since the electron temperature changes by a larger factor than the magnetic field. If the ions are also heated by the shock then the variation of R is correspondingly greater. As Gary and Biskamp (1971) have argued, Bernstein wave instability is the most likely explanation of the enhanced fluctuations observed in the Garching shock (Keilhacker *et al.* 1969). For the parameters of this shock we find R varies from about 3 at the front of the shock to about 15 at the rear.

We have also considered the case of oblique wave propagation, $\boldsymbol{k}.\boldsymbol{B} \neq 0$, where, as Gary (1970) has shown, the instability is ion acoustic. We find a similar result to equation (8)

$$R = 1 + \frac{3v_T}{2(v_d - \omega_R/k_y)}$$

but ω_R is here given by the ion acoustic frequency (Sanderson and Priest 1971). These results confirm the prediction of Woods (1969) that in strong shocks the large temperature gradient should play a dominant role. A fuller account of this work including a discussion of the ∇B terms will be given in a subsequent paper.

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Decay instability at the ion-sound frequency induced by a large amplitude Bernstein mode in a plasma

Abstract. Experimental results are presented which show that a large-amplitude wave of one of the Bernstein modes, above a certain threshold power, can decay into another Bernstein mode plus a low frequency ion-sound wave. At larger incident amplitudes, a whole spectrum of low-frequency ion waves was observed, with frequencies extending up to the ion plasma frequency. These results are compared with a previous theory and reasonable agreement is achieved.

Recently, the parametric excitation of various plasma waves has been of considerable interest both theoretically (Dubois and Goldman 1965, Silin 1965, Jackson 1967, Tzoar 1969, Pomeau 1967) and experimentally (Stern and Tzoar 1966, Hiroe and Ikegami 1967, Stern 1969, Porkolab and Chang 1969, Wong *et al.* 1970). In particular, the nonlinear coupling of high-frequency electric fields to low-frequency density